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ANNEX OF ALL APPENDICES TO

UNIVERSITY STUDENTS' KNOWLEDGE AND BIASES IN CONDITIONAL PROBABILITY REASONING

Carmen Díaz & Carmen Batanero

To article

APPENDICES

Appendices A and B are also contained in the paper;

Appendix C appears in the paper in different order in the text

ORDERED TABLES

APPENDIX A. ENGLISH TRANSLATION OF THE CPR TEST

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Item 1. (Estepa 1994). In a medical centre people were interviewed with the following results:

	55 years-old or younger	Older than 55	Total
Previous heart attack	29	75	104
No previous heart attack	401	275	676
Total	430	350	780

Suppose we select at random a person from this group:

a. What is the probability that the person had a heart attack
b. What is the probability that the person had a heart attack and, at the same time is older than 55?
c. When the person is older than 55, what is the probability that he/she had a heart attack?
d. When the person had a heart attack, what is the probability of being older than 55?

Item 2. (Tversky & Kahneman 1982a). A witness sees a crime involving a taxi in a city. The witness says that the taxi is blue. It is known from previous research that witnesses are correct 80% of the time when making such statements. The police also know that 15% of the taxis in the city are blue, the other 85% being green. What is the probability that a blue taxi was involved in the crime?

a. $\frac{80}{100}$ b. $\frac{15}{100}$ c. $\frac{15}{100}\times\frac{80}{100}$ d. $\frac{15\times 80}{85\times 20\ +\ 15\times 80}$

Item 3. (Sánchez 1996). A standard deck of playing cards has 52 cards. There are four suits (clubs, diamonds, hearts, and spades), each of which has thirteen cards (2, ... , 9, 10, *Jack*, *Queen*, *King*, *Ace*). We pick a card up at random. Let *A* be the event "*getting diamonds*" and *B* the event "*getting a Queen*". Are events *A* and *B* independent?

a. *A* and *B* are not independent, since there is the *Queen* of *diamonds*.
b. *A* and *B* are only then independent when we first get a card to see if it is a diamond, return the card to the pack and then get a second card to see if it is a *Queen*.
c. *A* and *B* are independent, since *P*(*Queen* of *diamonds*) = *P*(*Queen*) × *P*(*diamonds*).
d. The events *A* and *B* are not independent, since *P*(*Queen* | *diamonds*) ≠ *P*(*Queen*).

Item 4. There are four lamps in a box, two of which are defective. We pick up two lamps at random from the box, one after the other, without replacement. Given that the first lamp is defective, which answer is true?

a. The second lamp is more likely to be defective.
b. The second lamp is most likely to be correct.
c. The probabilities for the second lamp being either correct or defective are the same.

Item 5. (Eddy 1982). 10.3 % of women in a given city have a positive mammogram. The probability that a woman in this city has both positive mammogram and breast cancer is 0.8%. A mammogram given to a woman taken at random in this population was positive. What is the probability that she actually has breast cancer?

a. $\frac{0.8}{10.3} = 0.07767$, 7.77 %   b. $10.3 \times 0.8 = 8.24\,[\%]$, 8.24 %   c. 0.8 %

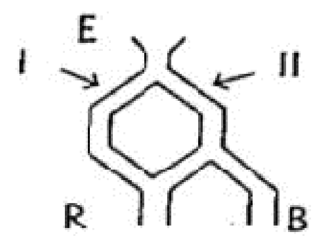
Item 6. (analogue to Tversky & Kahneman 1982 b). Suppose a tennis player reaches the Roland Garros final in 2005. He has to win 3 out of 5 sets to win the final. Which of the following two events is more likely or are they all equally likely?

a. The player will win the first set.
b. The player will win the first set but lose the match.
c. Both events a. and b. are equally likely.

Item 7. (Pollatsek, et al. 1987). A cancer test is administered to all the residents in a large city. A positive result is indicative of cancer and a negative result of no cancer. Which of the following results is more likely or are they all equally likely?

a. A person has in fact cancer supposed that he got a positive result.
b. To have a positive test supposed that the person has cancer.
c. The two events are equally likely.

Item 8. (Ojeda 1996).We throw a ball into the entrance E of a machine (see the figure). If the ball leaves the system through exit R, what is the probability that it passed through channel I?



a. 1/2
b. 1/3
c. 2/3
d. Cannot be computed

Item 9. (Falk 1986). Two black and two white marbles are put in an urn. We pick a marble from the urn. Then, without putting it back into the urn, we pick a second marble at random

9a. If the first marble is white, what is the probability that this second marble is white? $P(W_2 \mid W_1)$

i. 1/2
ii. 1/6
iii. 1/3
iv. 1/4

9b. If the second marble is white, what is the probability that the first marble is white? $P(W_1 \mid W_2)$

i. 1/3
ii. Cannot be computed
iii. 1/6
iv. 1/2

Item 10. An urn contains one blue and two red marbles. We pick two marbles at random, one after the other without replacement. Which of the events below is more likely or are they equally likely?

a. Getting two red marbles.
b. The first marble is red and the second is blue
c. The two events a) and b) are equally likely.

Item 11. Explain in your own words what a simple and a conditional probability are and provide an example.

Item 12. Complete the sample space in the following random experiments:

a. The gender (male/female) of the children in a three children family (e.g. $M\ F\ M$,...)
b. The gender of the children in a three children family if two or more children are male.

Item 13. In throwing two dice the product of the two numbers is 12.
What is the probability that none of the two numbers is a six (we take the order of the numbers into account)?

Item 14. 60% of the population in a city are men, 40% women. 50% of the men and 25% of the women smoke. We select a person from the city at random; what is the probability that this person is a smoker?

Item 15. A person throws a die and writes down the result (odd or even). It is a fair die (that is, all the numbers are equally likely). These are the results after 15 throws:

Odd, even, even, odd, odd, even, odd, odd, odd, odd, even, even, odd, odd, odd.

The person throws once more. What is the probability of getting an odd number this time?

Item 16. A group of students in a school take a test in mathematics and one in English. 80% of the students pass the mathematics test and 70% of the students pass the English test. Assuming that students' scores on the two tests are independent, what is the probability that a student passes both tests (mathematics and English)?

Item 17. According to a recent survey, 91% of the population in a city do lie and 36% of those lie about important matters. If we pick a person at random from this city, what is the probability that the person lies about important matters?

Item 18. (Totohasina 1982). Two machines M_1 and M_2 produce balls. Machine M_1 produces 40 % and M_2 60% of balls. 5% of the balls produced by M_1 and 1% of those produced by M_2 are defective. We take a ball at random and it is defective. What is the probability that that ball was produced by machine M_1?

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APPENDIX B.
SOLUTION AND COMPLEXITY ANALYSIS FOR OPEN-ENDED PROBLEMS

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Item 11. Explain in your own words what a simple and a conditional probability are and provide an example.

An intuitive definition of conditional probability could be:
"*Conditional probability* $P(A|B)$ *is the probability that an event A happens, given the occurrence of another event B*".
Another intuitive definition of conditional probability could be
"*number of joint occurrences of A and B, divided by the number of times that B has happened*".
A more formal definition is as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ where } P(B) > 0.$$

Item 12. Complete the sample space in the following random experiments:
a) The gender (male/female) of the children in a three children family (e.g. $M\,F\,M$,...)
b) The gender of the children in a three children family if two or more children are male.

In this item we assess the recognition that a conditional probability involves the restriction of the sample space. The correct answer in part a) requires the enumeration of all elements of the sample space. In part b), the student should reduce the sample space by the given condition as follows:

a) $E = \{(F\,F\,F), (F\,F\,M), (F\,M\,F), (M\,F\,F), (F\,M\,M), (M\,F\,M), (M\,M\,F), (M\,M\,M)\}$

b) $E^* = \{(F\,F\,M), (F\,M\,F), (M\,F\,F)\}$

Some students might not succeed to complete the enumeration in part a) and omit some cases; for example they might consider order is irrelevant. We consider partly correct those responses where the sample space *E* is not fully completed (for example, if the order is not considered), while the sample space E^* is correctly reduced from *E*.

Item 13. In throwing two dice the product of the two numbers is 12. What is the probability that none of the two numbers is a six (we take the order of the numbers into account)?

In this item we assess the student's competence to compute a conditional probability in the experiment "throwing two dice". In order to solve the problem correctly, the student must first define the correct sample space (36 possible cases), and then identify the cases where the product is 12 that define the restricted sample space: $\{(2, 6), (3, 4), (4, 3), (6, 2)\}$. Among these possible cases, only (3, 4) and (4, 3) have no six in it. Therefore the requested probability is equal to ½ since there are two favourable cases.

Item 14. Of the population in a city 60% are men; 40% are women. 50% of the men and 25% of the women smoke. We select a person from the city at random; what is the probability that this person is a smoker?

To solve this problem, the student could directly apply the total probability formula. Let S denote the event "the person smokes", M "the person is male" and F "the person is female", then:

$$P(S) = P(M \text{ and } S) + P(F \text{ and S}) = P(M) \times P(\text{S} \mid M) + P(F) \times P(S \mid F)$$
$$= 0.6 \times 0.5 + 0.4 \times 0.25 = 0.4$$

The student could also use a two way table to solve this problem. He could use proportional reasoning to compute the data in the different cells. Once the table cells are filled, the problem is reduced to a simple probability problem that can be solved with the Laplace's rule: 40 people smoke out of 100 people; the probability for a person taken at random smokes is 40/100.

	Does smoke	Doesn't smoke	Total
Male	30	30	60
Female	10	30	40
Total	40	60	100

Item 15. A person throws a die and writes down the result (odd or even). It is a fair die (that is, all the numbers are equally likely). These are the results after 15 throws:
Odd, even, even, odd, odd, even, odd, odd, odd, odd, even, even, odd, odd, odd.
The person throws once more. What is the probability to get an odd number this time?

To correctly solve this problem the student should understands that the previous occurrences do not affect the probability for the next outcome, since the successive throwing are independent. Therefore the probability to get an odd number in the next throwing is again ½. There are two possible cases (odd or even); only one of them (odd) is favourable. We consider it partly correct if the student provides a frequentist estimate for the probability (10/15) because he might have not assumed that the dice was fair. An incorrect solution is when students reason according the "gambler's fallacy" and think the probability of getting an odd number is now 5/15, since they expect the results should balance in the short run.

Item 16. A group of students in a school take a test in mathematics and one in English. 80% of the students pass the mathematics test and 70% of the students pass the English test. Assuming that students' scores on the two tests are independent, what is the probability that a student passes both tests (mathematics and English)?

In order to correctly solve this item, the students should apply the product rule, for the case of independent events. Let M denote the event "the student pass the mathematics test" and E the event "the student passes the English test". Then

$$P(M \cap E) = P(M) \times P(E) = 0.8 \times 0.7 = 0.56 \,.$$

Item 17. According to a recent survey, 91% of the population in a city do lie and 36% of those lie about important matters. If we pick a person at random from this city, what is the probability that the person lies about important matters?

This item assesses the student's competence to solve a product rule problem when the two events are dependent. Let *L* be the event "the person lies", and *I* the event "the person lies about an important matter", then,

$$P(L \cap I) = P(L) \times P(I \mid L) = 0.91 \times 0.36 = 0.3276 \,.$$

Item 18. (Totohasina 1992). Two machines M_1 and M_2 produce balls. Machine M_1 produces 40 % and M_2 60% of balls. 5% of the balls produced by M_1 and 1% of those produced by M_2 are defective. We take a ball at random and it is defective. What is the probability that that ball was produced by machine M_1?

In this item, we assess the student's competence to solve problems that involve Bayes' theorem. Let *D* the event "the ball is defective·, M_1 the event "the ball is produced by machine M_1 and M_2 the event "the ball is produced by machine M_2 then:

$$P(M_1 \mid D) = \frac{P(D \cap M_1)}{P(D)} = \frac{P(D \mid M_1) \times P(M_1)}{P(D)} \,.$$

The probability $P(D)$ can be computed by using the formula of the total probability:

$$P(D) = P(D \mid M_1) \times P(M_1) + P(D \mid M_2) \times P(M_2) = 0.05 \times 0.4 + 0.01 \times 0.6 = 0.026$$

Then, we can apply Bayes' formula, to compute the probability that the ball was produced by M_1 provided we take a defective ball, that is

$$P(M_1 \mid D) = \frac{0.02}{0.026} = 0.769$$

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APPENDIX C. COMPLEXITY ANALYSIS AND EXAMPLES OF CORRECT, PARTIALLY CORRECT, AND WRONG ANSWERS – ORDERED BY ITEMS

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The paper contains a complexity analysis of all items. It also contains typical examples of students' answers, which are classified (correct, partially correct, or wrong). In the paper, the examples are ordered by the category of the answer. Here the examples are ordered by items to alleviate the analysis of single items. The reader will find more examples of the study (in Spanish) in Díaz (2007, chapter 6).

Correct solutions

We consider correct those students who complete all the steps in the problem.

Partly correct solutions

We considered partly correct those students who correctly identify the data in a problem and the type of probability to be computed, but forget some data or make some mistakes in solving the problem.

Wrong responses

Some students provide a wrong solution or just provide a numerical wrong result for the problems with no justification.

Item 11. Explain in your own words what a simple and a conditional probability are and provide an example.

Complexity analysis. An intuitive definition of conditional probability could be:

"*Conditional probability* $P(A|B)$ *is the probability that an event A happens, given the occurrence of another event B*".

Another intuitive definition of conditional probability could be

"*number of joint occurrences of A and B, divided by the number of times that B has happened*".

A more formal definition is as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ where } P(B) > 0.$$

Correct student solution. In this item students have to define what conditional probability is and provide a correct example. In the following, the student uses a formula in order to define the conditional probability and provides correct examples.

"S*imple probability P*(*A*) *gives the probability for a single event.*

Conditional probability $P(A|B)$ *gives the probability of A provided that B happened.*

Examples:

Simple probability: probability that a person smokes; conditional probability; probability that a person smokes provided he is older than 50."

Partially correct solution.

"*Simple probability: when there is a single event.*

Conditional probability takes into account two events".

This answer is partly correct because in compound probability the two events also intervene; thus it is not clear that the student discriminates between compound and conditional probability.

"*Single probability is the probability that a variable happens*

while conditional probability is the probability that a variable happen when you fix a condition".

This response is imprecise as conditional probability can also be defined for events and not just for variables.

Wrong response.

"*Simple probability: getting a ball from an urn;*

conditional probability: getting a ball and then a second ball".

This response is incorrect because it lacks precision. The student only remembers an experiment given by the teacher concerning sampling balls from an urn, but he does not describe the event in full (for example specifying the colour of the ball, and the composition of the urn). Moreover, the example given for conditional probability corresponds to a compound probability instead of a conditional probability.

Item 12. Complete the sample space in the following random experiments:
a) The gender (male/female) of the children in a three children family (e.g. MFM ,...)
b) The gender of the children in a three children family if two or more children are male.

Complexity analysis. In this item we assess the recognition that a conditional probability involves the restriction of the sample space. The correct answer in part a) requires the enumeration of all elements of the sample space. In part b), the student should reduce the sample space by the given condition as follows:

a) $E = \{(FFF), (FFM), (FMF), (MFF), (FMM), (MFM), (MMF), (MMM)\}$

b) $E^* = \{(FFM), (FMF), (MFF)\}$

Some students might not succeed to complete the enumeration in part a) and omit some cases; for example they might consider order is irrelevant. We consider partly correct those responses where the sample space *E* is not fully completed (for example, if the order is not considered), while the sample space E^* is correctly reduced from *E*.

Correct student solution. Students should describe the restricted sample space in a conditional probability problem. An example of a correct answer is the following:

"*Gender of the children of a three children family*
$\{(MMM), (MMF), (MFM), (FMM), (MFF), (FMF), (FFM), (FFF)\}$
and gender of the children in a three children family if two or more children are male
$\{(MMM), (MMF), (MFM), (FMM)\}$ *"*

Partially correct solution.

a) $\{(MMM), (MMF), (MFF), (FFF)\}$;

b) $\{(MMF), (MMM)\}$ ".

Because of poor combinatorial reasoning, the student does not complete all possible events in the first part but gives a correct restriction of the sample space in the second.

Wrong response.

(MF, MF, MF) ".

The response is incorrect because the student only considers families with two children instead of families with three, and all the events are identical.

Item 13. In throwing two dice the product of the two numbers is 12. What is the probability that none of the two numbers is a six (we take the order of the numbers into account)?

Complexity analysis. In this item we assess the student's competence to compute a conditional probability in the experiment "throwing two dice". In order to solve the problem correctly, the student must first define the correct sample space (36 possible cases), and then identify the cases where the product is 12 that define the restricted sample space:

$\{(2, 6), (3, 4), (4, 3), (6, 2)\}$.

Among these possible cases, only (3, 4) and (4, 3) have no six in it. Therefore the requested probability is equal to ½ since there are two favourable cases.

Correct student solution. In this item students need to solve a conditional probability problem in a single experiment. In the following correct response, the student uses the Laplace rule to solve the problem, after a correct identification of the favourable and possible cases:

$\{(2,6),(3,4),(6,2),(4,3)\}, \quad 2/4 = 1/2 = 0.5$".

Partially correct solution.

"*Possible cases* = 36; *combinations:* $(2\times 6),(6\times 2),(3\times 4),(4\times 3)$;

the probability is therefore = 2/36 = 1/18"

The student correctly identifies the possible cases in the compound experiment and all the favourable cases for the product 12. As none of the numbers can be 6, he makes a correct restriction of the sample space; he considers only 2 favourable cases in applying the Laplace rule. However, he mistakes the denominator in the Laplace rule by considering the favourable cases in the unrestricted sample space; thus he obtains a wrong probability.

Wrong response.

" $\textit{Possible cases} = 12, \quad \textit{favourable cases} = 2, \quad P = 2/12 = 1/6$".

The student is unable to find the sample space in the compound experiment; instead she adds the number of events in each sample space for the single experiment. Moreover, she computes the simple probability that none of the two terms in the product is 6 rather than a conditional probability.

Item 14. Of the population in a city 60% are men; 40% are women. 50% of the men and 25% of the women smoke. We select a person from the city at random; what is the probability that this person is a smoker?

Complexity analysis. To solve this problem, the student could directly apply the total probability formula. Let *S* denote the event "the person smokes", *M* "the person is male" and *F* "the person is female", then:

$$\begin{aligned} P(S) &= P(M \text{ and } S) + P(F \text{ and } \mathrm{S}) = P(M) \times P(\mathrm{S} \mid M) + P(F) \times P(S \mid F) \\ &= 0.6 \times 0.5 + 0.4 \times 0.25 = 0.4 \end{aligned}$$

The student could also use a two way table to solve this problem. He could use proportional reasoning to compute the data in the different cells. Once the table cells are filled, the problem is reduced to a simple probability problem that can be solved with the Laplace's rule: 40 people smoke out of 100 people; the probability for a person taken at random smokes is 40/100.

	Does smoke	Doesn't smoke	Total
Male	30	30	60
Female	10	30	40
Total	40	60	100

Correct student solution. Students are asked to apply the total probability rule. In the next example, the student correctly identifies the events and their probabilities and applies the total probability formula:

$$P(S) = \frac{60 \times 50}{100^2} + \frac{40 \times 25}{100^2} = 0.3 + 0.1 = 0.4 \text{''}.$$

Partially correct solution.

The following student identifies the number of men and women and adds these numbers to get the total of people smoking. However, in applying the total probability rule he assumes the same proportion of men and women in the population, which is incorrect:

200 *people;* 50 *men smoke and* 35 *women smoke; a total of* 85 *among* 200 *smoke;*
P(*smoking*) = 85/200*".*

Wrong response.

130".

The student gives a response that is not related to the problem data.

Item 15. A person throws a die and writes down the result (odd or even). It is a fair die (that is, all the numbers are equally likely). These are the results after 15 throws:

Odd, even, even, odd, odd, even, odd, odd, odd, odd, even, even, odd, odd, odd.

The person throws once more. What is the probability to get an odd number this time?

Complexity analysis. To correctly solve this problem, the student should understand that the previous occurrences do not affect the probability for the next outcome, since the successive throwing are independent. Therefore the probability to get an odd number in the next throwing is again ½. There are two possible cases (odd or even); only one of them (odd) is favourable. We consider it partly correct if the student provides a frequentist estimate for the probability (10/15) because he might have not assumed that the dice was fair. An incorrect solution is when students reason according the "gambler's fallacy" and think the probability of getting an odd number is now 5/15, since they expect the results should balance in the short run.

Correct student solution. Students need to solve a conditional probability problem, in case the events are independent. In this example, the student identifies the independence in the successive throwing of the dice:

"$1/2$ *as the result does not depend on previous results*".

Partially correct solution.

"10/15".

The student correctly identifies independence of events; however he assumes that the die is biased (in spite of what is said in the problem statement). He gives a correct frequentist estimate of probability, but the solution is partly correct, since he assumed the die is biased.

Wrong response.

Some students assume that, after getting a majority of odd results in throwing a die, the probability of even result increases.

We consider this response as incorrect, since this is an inaccurate perception of equal likelihood and independence; the student is reasoning according to the representativeness heuristics (Kahneman, Slovic; & Tversky 1982).

According to this heuristic, when people are asked to judge or calculate the probability that an object or event *A* belongs to class or process *B*, probabilities are evaluated by the degree to which *A* is representative of *B*, that is, by the degree to which *A* resembles *B*.

As 10 odd results out of 15 do not seem to be a representative sample for outcomes coming from a fair die, so students in this category assume there is more likelihood to get an even result after getting 10 odd results out of 15, which seems more representative.

Item 16. A group of students in a school take a test in mathematics and one in English. 80% of the students pass the mathematics test and 70% of the students pass the English test. Assuming that students' scores on the two tests are independent, what is the probability that a student passes both tests (mathematics and English)?

Complexity analysis. In order to correctly solve this item, the students should apply the product rule, for the case of independent events. Let M denote the event "the student pass the mathematics test" and E the event "the student passes the English test". Then

$$P(M \cap E) = P(M) \times P(E) = 0.8 \times 0.7 = 0.56 .$$

Correct student solution. Students have to apply the product rule in the case of independence of events. In this response, the student correctly identifies the data and the independence of experiments and then applies the product rule for independent events:

"$P(A_1 \cap A_2) = P(A_1) \times P(A_2) = 0.8 \times 0.7$".

Partially correct solution.

"$P(Math \cap English) = 80/100 + 70/100 = 150/100$".

This student correctly identifies the data. But he confuses the rules for computing the probability of the union and intersection of events. Moreover he does not realise that a probability cannot be higher than 1.

Wrong response.

Some students enumerated all the cases in throwing two dice (36 cases), but were unable to continue the problem.

Item 17. According to a recent survey, 91% of the population in a city do lie and 36% of those lie about important matters. If we pick a person at random from this city, what is the probability that the person lies about important matters?

Complexity analysis. This item assesses the student's competence to solve a product rule problem when the two events are dependent. Let L be the event "the person lies", and I the event "the person lies about an important matter", then,

$$P(L \cap I) = P(L) \times P(I \mid L) = 0.91 \times 0.36 = 0.3276 .$$

Correct student solution. Students have to apply the product rule in the case of dependence of events. This student correctly identifies the data, the dependence of experiments, and then applies the product rule for dependent events:

"$P(\textit{Lying about important matters}) = 0.91 \times 0.36 = 0.3276$".

Partially correct solution.

"$P(\textit{Lying} \mid \textit{important things}) = 0.36 / 0.91 = 0.40$".

In this example the student identifies the data, but he computes a conditional probability instead of a joint probability.

Wrong response.

"55%".

This response has no relationship to the problem data and the students do not provide reasoning for the same.

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Manfred Borovcnik & Ramesh Kapadia (Eds)

ORDERED TABLES 2 AND 3 – ORDERED BY MAGNITUDE OF SUCCESS OF TEACHING

% correct	Teaching		
Item	No n = 177	Yes n = 206	p -value
1a. Simple probability, from a 2- way table	35	69	0.000
1b. Computing conditional probability from a 2- way table	67	94	0.000
1c. Computing joint probability, from a 2- way table	29	63	0.000
1d. Computing inverse conditional probability from a 2- way table	37	70	0.000
2. Base rate fallacy	33	53	0.000
3. Independence /mutually exclusiveness	23	41	0.000
4. Solving a conditional probability problem, in case of dependence	77	89	0.001
5. Computing conditional probability from joint & compound probability	37	48	0.042
6. Conjunction fallacy	21	24	0.465
7. Transposed conditional /causal-diagnostic	35	35	0.989
8. Time axis fallacy	8	13	0.142
9a. Computing conditional probability, dependence	72	81	0.050
9b. Time axis fallacy	37	25	0.009
10. Solving a joint probability problem in diachronic experiments	62	76	0.002
11. Defining conditional probability and giving an example	10	25	0.000
12. Describing the restricted sample space	46	64	0.050
13. Solving a conditional probability problem, in a single experiment	20	35	0.005
14. Solving total probability problem	18	69	0.000
15. Solving a conditional probability problem, for independent events	35	60	0.000
16. Solving product rule problem for two independent events	26	49	0.000
17. Solving product rule problem for two dependent events	24	62	0.000
18. Solving Bayes' problem	4	50	0.000
Average	34,4	54,3	

Table 2a. Percentages of correct responses to items in students with (n=177) and without (n=206) instruction.

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% correct Item	Teaching No n = 177	Teaching Yes n = 206	p-value	Difference in success rates	Interpretation of the effect of teaching
9b. Time axis fallacy	37	25	0.009	-12	Teaching did not help to overcome part of the biases in conditional probability reasoning in these students. Probably teaching of the topic has to confront the students directly with these biases, since formal teaching alone is not enough.
7. Transposed conditional /causal-diagnostic	35	35	0.989	0	
6. Conjunction fallacy	21	24	0.465	3	
8. Time axis fallacy	8	13	0.142	5	
9a. Computing conditional probability, dependence	72	81	0.050	9	These items were easy even without instruction. Three of these problems are about sampling with and without replacement and students could apply Laplace rule. The context was familiar; students in psychology are familiar with diagnosis situations; moreover the formula was given to the students.
5. Computing conditional probability from joint & compound probability	37	48	0.042	11	
4. Solving a conditional probability problem, in case of dependence	77	89	0.001	12	
10. Solving a joint probability problem in diachronic experiments	62	76	0.002	14	
11. Defining conditional probability and giving an example	10	25	0.000	15	Remembering the precise definition is a very formal task, even for students with instruction; this task was very hard.
13. Solving a conditional probability problem, in a single experiment	20	35	0.005	15	This problem was quite difficult; it deals with dependence in *simple* experiments; for students it is easier to see that two events are dependent with compound experiments. This suggests to use more examples of conditional probablity in *simple* experiments (now this is not very common).
3. Independence /mutually exclusiveness	23	41	0.000	18	This item is related to one of the biases (confounding mutually exclusivity with independence; teaching did not help much in this bias although the situation is better than for the other biases.
12. Describing the restricted sample space	46	64	0.050	18	Again a very formal item. But in this case instruction helped a little more, not too much because not much time is devoted to describe sample spaces.
2. Base rate fallacy	33	53	0.000	20	This item is related to one of the biases (time axis fallacy) teaching did not help much in this bias although the situation is better than for the other biases.
16. Solving product rule problem for two independent events	26	49	0.000	23	The difficulty was moderate even without instruction; instruction improved results but the effect is not so high because these items had moderate difficulty.
15. Solving a conditional probability problem, for independent events	35	60	0.000	25	
1b. Computing conditional probability from a 2- way table	67	94	0.000	27	Reading 2x2 tables is considered to be an intuitive competence. However, some of the tasks were difficult or moderate in difficulty. Instruction helped improving in all the cases because 2x2 tables were also used as a tool to solve the probability problems in teaching.
1d. Computing inverse conditional probability from a 2- way table	37	70	0.000	33	
1c. Computing joint probability, from a 2- way table	29	63	0.000	34	
1a. Simple probability, from a 2- way table	35	69	0.000	34	
17. Solving product rule problem for two dependent events	24	62	0.000	38	Problems that were not intuitive without instruction. These problems involve several steps and mixed the different types of probabilities: simple, compound, conditional in situations where the events are dependent from another one.
18. Solving Bayes' problem	4	50	0.000	46	
14. Solving total probability problem	18	69	0.000	51	
Average	34,4	54,3		20,0	

Table 2b. Ordered by magnitude of effect of teaching (difference of success rates).

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	Teaching						Difference	
	No			Yes				
Item	Wrong	Partly	Com-plete	Wrong	Partly	Com-plete	Chi-Square	*p*-value
11. Defining conditional probability	53,1	37,3	**9,6**	33,5	41,3	**25,2**	28,8	0,000
12. Describing the restricted sample space	30,5	23,2	**46,3**	15,5	20,9	**63,6**	20,4	0,000
13. conditional probability problem, single experiment	54,8	24,9	**20,3**	46,6	18,9	**34,5**	9,6	0,008
14. total probability problem	43,5	38,4	**18,1**	11,7	18,9	**69,4**	104,5	0,000
15. conditional probability, in case of independence	36,2	29,4	**34,5**	23,3	10,7	**66,0**	41,0	0,000
16. product rule problem, in case of independence	46,3	28,2	**25,4**	24,3	26,7	**49,0**	27,4	0,000
17. product rule problem, in the case of dependence	44,6	31,6	**23,7**	18,4	19,9	**61,7**	57,6	0,000

Table 3a. Percentages of incorrect, partly correct, and correct solutions to items in students with and without instruction.

Wrong = Blank or wrong; partly = partly correct solution; complete = completely correct solution

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Item	Teaching: No – Wrong	No – Partly	No – Complete	Yes – Wrong	Yes – Partly	Yes – Complete	Differences: Chi-Square	p-value	Wrong	Partly	Complete
11. Defining conditional probability	53,1	37,3	**9,6**	33,5	41,3	**25,2**	28,8	0,000	-19,6	4,0	**15,6**
14. total probability problem	43,5	38,4	**18,1**	11,7	18,9	**69,4**	104,5	0,000	-31,8	-19,5	**51,3**
13. conditional probability problem, single experiment	54,8	24,9	**20,3**	46,6	18,9	**34,5**	9,6	0,008	-8,2	-6,0	**14,2**
17. product rule problem, in the case of dependence	44,6	31,6	**23,7**	18,4	19,9	**61,7**	57,6	0,000	-26,2	-11,7	**38,0**
16. product rule problem, in case of independence	46,3	28,2	**25,4**	24,3	26,7	**49,0**	27,4	0,000	-22,0	-1,5	**23,6**
15. conditional probability, in case of independence	36,2	29,4	**34,5**	23,3	10,7	**66,0**	41,0	0,000	-12,9	-18,7	**31,5**
12. Describing the restricted sample space	30,5	23,2	**46,3**	15,5	20,9	**63,6**	20,4	0,000	-15,0	-2,3	**17,3**

Table 3b. Ordered by correct solutions without teaching (difficulty of items).

Wrong = Blank or wrong; partly = partly correct solution; complete = completely correct solution

Item	Differences: p-value	Chi-Square	Wrong	Complete	Ranks for differences in the sense of instruction is better: Chi-Square	Wrong	Complete	Mean ranks
11. Defining conditional probability	0,000	28,8	-19,6	**15,6**	4	4	**6**	4,7
14. total probability problem	0,000	104,5	-31,8	**51,3**	1	1	**1**	1
13. conditional probability problem, single experiment	0,008	9,6	-8,2	**14,2**	7	7	**7**	7
17. product rule problem, in the case of dependence	0,000	57,6	-26,2	**38,0**	2	2	**2**	2
16. product rule problem, in case of independence	0,000	27,4	-22,0	**23,6**	5	3	**4**	4
15. conditional probability, in case of independence	0,000	41,0	-12,9	**31,5**	3	6	**3**	4
12. Describing the restricted sample space	0,000	20,4	-15,0	**17,3**	6	5	**5**	5,3

Table 3b. Ordered by correct solutions without teaching (difficulty of items) - continued.

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Item	Differences Mean ranks	Interpretation
11. Defining conditional probability	4,7	This is a very formal item, in spite that we asked students to use their own words and for this reason there is a lot of missing responses, even after instruction. Defining and giving a correct example was easier in both groups (these are the students whose answers are classified as partly correct). After the instruction 15.6% more students were able to explain and give a correct example of conditional probability.
14. total probability problem	1,0	Solving a total probability problem involves dealing with many mathematical concepts: simple, conditional and compound probability, partition, intersection and union of events. This explains the dramatic effect of teaching, because students practiced the solving of other similar problems with the help of tree diagrams that proved to be a good tool to help them identify the data and recognize the different types of probabilities involved.
13. conditional probability problem, single experiment	7,0	This problem was quite difficult because it deals with conditional probability in a case of dependence and in a simple experiment and for students it is easier to see that two events are dependent when we deal with a compound experiment. This suggests the need to use more examples of conditional probability in *simple* experiments (while in our teaching these examples are not too common).
17. product rule problem, in the case of dependence	2,0	The effect of teaching was high; students seemed to grasp well the use of the product rule, both in case of dependence and independence (see next item). In this particular item, the context helped the students to apply the product rule because dependence is very clear in the situation.
16. product rule problem, in case of independence	4,0	Again, the teaching helped students learn the use of the product rule. From their own experience in examinations, the students can easily perceive independence.
15. conditional probability, in case of independence	4,0	The item was not too difficult even for students without instruction, although there was a clear improvement in the instruction group. Instruction seemed to help overcome the belief in the law of small numbers in this item.
12. Describing the restricted sample space	5,3	Item of moderate difficulty even without teaching; students applied their previous knowledge of simple probability to this item.

Table 3c. Ordered by correct solutions without teaching (difficulty of items) - Interpretation.

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