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“Research and Developments in Probability Education”**

Manfred Borovcnik & Ramesh Kapadia (Eds)

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1. INTRODUCTION

Research and Developments in Probability Education

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Manfred Borovcnik & Ramesh Kapadia

ABSTRACT. In the topic study group on probability at ICME 11 a variety of ideas on probability education were presented. Some of the papers have been developed further by the driving ideas of interactivity and use of the potential of electronic publishing. As often happens, the medium of research influences the results and thus – not surprisingly – the research change its character during this process. This paper provides a summary of the main threads of research in probability education across the world and the result of an experiment in electronic communication. For convenience of international readers, abstracts in Spanish and German have been supplied, as well as hints for navigation to linked electronic materials.

KEYWORDS. Research, probability education, ICME 11, electronic publishing, interactivity.

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2. CONDITIONAL PROBABILITY AND BAYES' THEOREM

University Students' Knowledge and Biases

in Conditional Probability Reasoning

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Carmen Díaz & Carmen Batanero

ABSTRACT. The research question in this study was assessing possible relationships between formal knowledge of conditional probability as well as biases related to conditional probability reasoning: fallacy of the transposed conditional; fallacy of the time axis; base rate fallacy; synchronic and diachronic situations; conjunction fallacy; and confusing independence and mutually exclusiveness. Two samples of university students majoring in psychology and following the same introductory statistics course were given the CPR test before ($n = 177$) and after ($n = 206$) formal teaching of conditional probability. Results indicate a systematic improvement in formal understanding of conditional probability and in problem solving capacity but little change in those items related to psychological biases.

KEYWORDS. Conditional probability, biases, instruction.

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On Conditional Probability Problem Solving Research

– Structures and Contexts

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M. Pedro Huerta

ABSTRACT. In this paper we summarize the research we have recently carried out on classifying problems of conditional probability. We investigate a particular world of school word problems we call ternary problems of conditional probability. With the help of a mathematical object, the trinomial graph, and the analysis and synthesis method, we propose a framework for a structural, didactical and phenomenological analysis of the ternary problems of conditional probability. Consequently, we have organized this world into several types of problems. With respect to students' behaviour, we identify four types of thinking processes related to data format and the use of data. We also illustrate our approach by use of the diagnostic test situation, and in the particular context of health.

The main purpose of our work is to improve secondary school students' understanding of conditional probability and their probability literacy by proposing a teaching approach based on problem solving within appropriate contexts. We believe that the framework we present in this paper could help teachers and researchers in this purpose.

KEYWORDS. Conditional probability, problem solving.

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3. THE SCHOOL PERSPECTIVE: PRE- AND MISCONCEPTIONS

A Student's Synthesis of Tacit and Mathematical Knowledge as a Researcher's Lens on Bridging Learning Theory

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Dor Abrahamson

ABSTRACT. What instructional materials and practices will help students make sense of probability notions? Li (11 years) participated in an interview-based implementation of a design for the binomial. The design was centered around an innovative urn-like random generator, creating opportunities to reconcile two mental constructions of anticipated outcome distributions: (a) holistic perceptual judgments based in tacit knowledge of population-to-sample relations and implicitly couched in terms of the aggregate events with no attention to permutations on these combinations; and (b) classicist-probability analytic treatment of ratios between the subset of favorable to all elemental events with attention to the permutations. We argue that constructivist and sociocultural perspectives on mathematics learning can be reconciled by revealing interactions of intuitive and formal resources in individual development of deep conceptual understanding. Learning is the guided process of blending two constructions of problematized situations: the phenomenologically immediate and the semiotically mediated.

KEYWORDS. Design-based research, cognitive science, sociocultural theory, cultural semiotics, binomial, combinatorial analysis, sample space.

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Hands-On Activities for Fourth Graders:

A Tool Box for Decision-Making and Reckoning with Risk

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Laura Martignon & Stefan Krauss

ABSTRACT. The intention of this work is to exhibit how children can be provided with a kit of elementary tools for judgment under uncertainty, for good decision making and for reckoning with risk. Children, we claim, can acquire this tool kit through a mosaic of simple, play-based activities which are devised to make them aware of the characteristics of uncertainty. We present a sequence of tasks that build upon each other, beginning with the Wason selection task, moving on to probabilistic tasks, tasks in elementary Bayesian reasoning comparing proportions and, finally, to comparing risks. This research is guided and inspired by empirical results on human decision making in the medical and financial domain.

KEYWORDS. Tool kit, Tinker Cubes, Bayesian reasoning, Wason's experiment, risk.

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Recency Effects in Primary-Age Children and College Students

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Francesca Chiesi & Caterina Primi

ABSTRACT. We investigate the evolution of probabilistic reasoning with age and some related biases, such as the negative/positive recency effects. Primary school children and college students were presented with probability tasks in which they were asked to estimate the likelihood of the next occurring event after a sequence of independent outcomes. Results indicate that older children perform better than younger children and college students. Concerning biases, the positive recency effect decreases with age whereas no age-related differences are found for the negative recency effect. Theoretical and educational implications of results are discussed.

KEYWORDS. Probabilistic reasoning, recency effects, primary school children, college students.

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4. THE TEACHERS' PERSPECTIVE: PRE- AND IN-SERVICE COURSES

A Practical Approach to Probability in the Context of a Science Fair

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Hugo M. Hernández Trevethan, Verônica Y. Kataoka, & Marcelo da Silva Oliveira

ABSTRACT. In a society that generates information rapidly, schools have to fulfil their programmes imaginatively. Thus, extra-curricular activities may be helpful for the students to acquire wider knowledge than that they may get within the classrooms. On the other hand, randomness is present in almost all everyday decisions, mainly based on prior information so it is important to have at least a rough idea on how specific events may affect the chances of other events. We explore both ideas here in the context of a science fair, in which two high-school senior students conducted an investigation about conditional probability using a game called “Shut the box”. We also want to pose, as a research question, if, after their participation in the science fair, these students have reached higher levels in probabilistic reasoning compared to their classmates or have acquired knowledge about concepts far beyond the official curriculum.

KEYWORDS. Science Fair, Conditional Probability, Bayes' theorem, extra-curricular work, project-based work, personal development.

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Parallel Discussion of Classical and Bayesian Ways as an Introduction to Statistical Inference

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Ödön Vancsó

ABSTRACT. The purpose of this paper is to report on the conception and some results of a long-term university research project in Budapest. The study is based on an innovative idea of teaching the basic notions of classical and Bayesian inferential statistics parallel to each other to teacher students. Our research is driven by questions like: Do students understand probability and statistical methods better by focussing on subjective and objective interpretations of probability throughout the course? Do they understand classical inferential statistics better if they study Bayesian ways, too? While the course on probability and statistics has been avoided for years, the students are starting to accept the “parallel” design. There is evidence that they understand the concepts better in this way. The results also support the thesis that students’ views and beliefs on mathematics decisively influence work in their later profession. Finally, the design of the course integrates reflections on philosophical problems as well, which enhances a wider picture about modern mathematics and its applications.

KEYWORDS. Bayesian statistics, favourable relation, statistical inference, confidence interval, Bayesian regions of highest posterior density (RHD).

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5. IMPACT OF TECHNOLOGY

Shaping the Experience of Young and Naïve Probabilists

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Dave Pratt & Ramesh Kapadia

ABSTRACT. This paper starts by assessing deficiencies in teaching statistics before summarizing research that has focused on pupils' misconceptions of probability. In contrast, in previous research has explored what pupils of age 11-12 years do know and can construct, given access to a carefully designed environment. These pupils judged randomness according to unpredictability, lack of pattern in results, lack of control over outcomes and fairness, as indeed would experts. However, it was only through interaction with a virtual environment, ChanceMaker that the pupils began to express situated meanings for aggregated long-term randomness. That data is then re-analyzed in order to reflect upon the design decisions that shaped the environment itself. Four main design heuristics are identified and elaborated: testing personal conjectures, building on pupil knowledge, linking purpose and utility, fusing control and representation. It is conjectured that these heuristics are of wider relevance to teachers and lecturers, who aspire to shape the experience of young and naïve probabilists through their actions as designers of tasks and pedagogical settings.

KEYWORDS. ChanceMaker, design, technology, micro-world.

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Building a Connection between Experimental and Theoretical Aspects of Probability

Seth Ireland & Jane Watson

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ABSTRACT. This paper addresses a question identified by Graham Jones: what are the connections made by students in the middle years of schooling between classical and frequentist orientations to probability? It does so based on two extended lessons with a class of Grade 5/6 students and in-depth interviews with eight students from the class. The Model 1 version of the software TinkerPlots was used in both settings to simulate increasingly large samples of random events. The aim was to document the students' understanding of probability on a continuum from experimental to theoretical, including consideration of the interaction of manipulatives, the simulator, and the law of large numbers. A cognitive developmental model was used to assess students' understanding and recommendations are made for classroom interventions.

KEYWORDS. Experimental estimates of probability, Theoretical probability, simulation, law of large numbers, *TinkerPlots*.

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6. EPILOGUE

Chance Encounters – 20 Years Later

Fundamental Ideas in Teaching Probability at School Level

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Ramesh Kapadia

ABSTRACT. This paper considers how probability is now taught in England and the way that the curriculum reflects key research ideas from the last few decades. Links are made to work undertaken in probability education and the way that challenges in the book, *Chance Encounters*, have been met. This is based on the current curriculum and also the performance of children in tests. The key question considered is the extent to which the teaching of probability has changed over the last twenty years. The conclusion notes that there is some way to go in ensuring children are well versed in probability.

KEYWORDS. Probability, secondary school, teaching.

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